$$\therefore \quad f(x) = f(x_0) + (x - x_0)\, f(x, x_0) \quad \to ①$$

$||^{rly}$ $$f(x, x_0, x_1) = \frac{f(x, x_0) - f(x_0, x_1)}{x - x_1}$$

$$\therefore \quad f(x, x_0) = f(x_0, x_1) + (x - x_1)\, f(x, x_0, x_1)$$

Using this value of $f(x, x_0)$ in ①, we have

$$f(x) = f(x_0) + (x - x_0) \cdot f(x_0, x_1) + (x - x_1)\, f(x, x_0, x_1) \quad \to ②$$

Again

$$f(x, x_0, x_1, x_2) = \frac{f(x, x_0, x_1) - f(x_0, x_1, x_2)}{x - x_2}$$

$$\therefore \quad f(x, x_0, x_1) = f(x_0, x_1, x_2) + (x - x_2)\, f(x, x_0, x_1, x_2) \quad \to ③$$

Continuing in this manner, we get

$$\begin{aligned} f(x) = {} & f(x_0) + (x - x_0)\, f(x_0, x_1) + (x - x_0)(x - x_1)\, f(x_0, x_1, x_2) \\ & + (x - x_0)(x - x_1)(x - 2)\, f(x_0, x_1, x_2, x_3) + \cdots \\ & + (x - x_0)(x - x_1)(x - 2) \ldots (x - x_{n-1})\, f(x_0, x_1, \ldots x_n) \\ & + (x - x_0)(x - x_1)(x - x_2) \ldots (x - x_n)\, f(x_0, x_1, \ldots x_n) \end{aligned} \quad \to ④$$

If $f(x)$ is a polynomial of degree n, then

$$f(x, x_0, x_1, \ldots, x_n) = 0 \qquad (\because (n+1)^{th} \text{ difference})$$

Hence ④ becomes,

$$\begin{aligned} f(x) = {} & f(x_0) + (x - x_0)\, f(x_0, x_1) + (x - x_0)(x - x_1)\, f(x_0, x_1, x_2) \\ & + \cdots + (x - x_0)(x - x_1) \cdots (x - x_{n-1})\, f(x_0, x_1, \ldots, x_n) \end{aligned} \quad \to ⑤$$

Equation ⑤ is called NDDF for unequal intervals

Pb 1. Using Newton's divided diff. formula, find the values of $f(2), f(8)$ and $f(15)$ given the following table.

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028.

Sol: We form the divided diff. table since the intervals are unequal.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	48				
		$\frac{100-48}{5-4} = 52$			
5	100		$\frac{97-52}{7-4} = 15$		
		$\frac{294-100}{7-5} = 97$		$\frac{21-15}{10-4} = 1$	
7	294		$\frac{202-97}{10-5} = 21$		0
10	900	$\vdots$		$\frac{27-21}{11-5} = 1$	
$\vdots$	$\vdots$				0
				$\frac{33-27}{13-7} = 1$	

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + \cdots \rightarrow (1)$$

Here $x_0 = 4$; $x_1 = 5$, $x_2 = 7$; $x_3 = 10$, $x_4 = 11$, $x_5 = 13$ and

$f(x_0) = 48$; $f(x_0,x_1) = 52$, $f(x_0,x_1,x_2) = 15$, $f(x_0,x_1,x_2,x_3) = 1$.

Hence using these in (1), we have.

$$f(x) = 48 + (x-4)52 + (x-4)(x-5)15 + (x-4)(x-5)(x-7)1$$

$\therefore$ $f(2) = 4$

$f(8) = 448$

$f(15) = 3150$.